

## A- Arithmetic Sequence

Time limit: 3 Secs
Incremental Arithmetic Sequence is a sequence of integers such that the difference between the consecutive terms is constant and positive (e.g. the sequence $5,7,9,11,13,15$ with common difference of 2). You need to count the number of arithmetic sequences in the set $N=\{1,2, \ldots, n\}$.

## Input

Each test case is represented by a line containing a positive integer $n$ less than 1,000,001.

## Output

For each $n$, print the number of arithmetic sequences with at least two elements mod $10^{9}+7$.

|  | Sample Input | Sample Output |
| :--- | :--- | :--- |
| 2 | 1 |  |
| 3 |  | 4 |
| 4 | 9 |  |
| 1000 | 3281346 |  |



## B- Big Password

Time limit: 5 Secs

You know that, nowadays, in each website a pair of username and password is required (e.g. Sharecode). However, I have a bad habit!!! I can only choose the passwords, which are integers generated by concatenation of the sequence of the numbers 1 to n . In addition, the generated big integer should be divisible by ( $n+1$ ) (e.g. $12 \operatorname{Mod} 3=0$ and $12345678 \operatorname{Mod} 9=0$ ). I like the values of $n$, which can generate such passwords for me. In order to convince me to break that habit; you are going to show me how many passwords are there for values of $n$ not greater than N.

## Input

The input is started by an integer $0<t \leq 20$ followed by $t$ test cases. Each test case is represented by one line containing the integer $N \leq 100,000$.

## Output

For each test case, output in a separate line, the number of passwords can be generated by all values of $n \leq N$.

|  | Sample Input | Sample Output |
| :--- | :--- | :--- |
| 4 | 0 |  |
| 1 | 2 |  |
| 10 | 4 |  |
| 100 | 4 |  |
| 1000 |  |  |



## C - Compression

## Time limit: 6 Secs

Our university researchers are interested in a particular string coding method. Consider a string of lowercase English letters ('a'-'z'). According to this coding method one can select a prefix of the string and optionally replace its non-overlapping occurrences in the string each with ' P ' and its reverse occurrences in the string each with 'R'. The prefix itself is not replaced and is followed by ' $X$ ' only if any replacements have taken place already. As an example consider the string 'abaaabaabaaba'. The string has different representations with the prefix selected, among which are 'abaaabaabaaba', 'abaXaPabaP', and 'abaaXPbR'. As you can see, the coded string can easily be decoded back to its original form. Our researchers want to know the effectiveness of this coding as a compression technique.

Having the original string at hand you are asked to help us know the shortest possible coding that can be created using this method.

## Input

The first line of the input contains an integer $T \leq 100$ denoting the number of test-cases. Each test-case is a string of length of at most 25,000 provided on a separate line.

## Output

For each test-case, output the shortest possible length of the code in a line.

| Sample Input | Sample Output |
| :--- | :--- |
| 2 | 8 |
| abaaabaabaaba | 7 |
| abcdabcda |  |



## D- Division

Time limit: 10 Secs

Mohammad has made a simple calculator for basic operations (i.e. Summation, Subtraction, Multiplication and Division). Most of the challenges were in implementation of Division. This is why; he decided to implement only integer-division such that the final result is also integer (floor of the real result). For example, $5 / 2$ is equal to 2 . However, implementing even this division is hard for Mohammad. He thought that "For large numbers, no one can understand how much the answer is wrong"!!! Hence, he used a simple trick to solve this hard problem. You know that implementation of division by 2 is very simple. So for dividing A by B (i.e. floor of $A / B)$, he divides both of $A$ and $B$ by 2 until $B$ becomes 1 . Then, $A$ is the final answer. For example, $\frac{100}{20}=\frac{50}{10}=\frac{25}{5}=\frac{12}{2}=\frac{6}{1}=6$. If $A$ and $B$ are in the ranges [ $n M i n, n M a x$ ] and [mMin, mMax], respectively, would you please help Mohammad to know, how many pairs of $(A, B)$ are there for which, his computer correctly computes the division result?

## Input

The input is started by an integer $0<t \leq 100$ followed by $t$ testcases. Each testcase is represented by one line containing four integers $n M i n, n M a x, m M i n$ and $m$ Max separated by space ( $0 \leq n M i n \leq n M a x \leq 100,000$ and $0<m M i n \leq m M a x \leq 10,000$ ).

## Output

For each testcase, print the output in a separate line. The output is the number of pairs in the given range with a correct division-result by use of Mohammad's calculator.

| Sample Input | Sample Output |
| :---: | :---: |
| 3 | 22 |
| 01012 | 19204721 |
| 123456789012345678 | 157192354 |
| 0100000110000 |  |



## E- Extra Point

Time limit: 3 Secs

Dr. MT is a faculty member of Sh. University and teaches the undergraduate students "Fundaments of Programming". He likes to encourage his students to compete in weekly contests held in the university. Hence, he decided to assign an extra point to each student who competes in these contests according to the following rules:

- The student should compete in all contests.
- Extra point is assigned to the student only for valid contests, in which the number of solved problems is not less than the ones of the previous contests.
- Extra point of each valid contest is as much as the number of solved problems.
- Overall extra point is the sum of extra points achieved in the contests.

At the first day, one of the students knows how many problems he can solve in each contest. However, the maximum extra point is not certainly achieved by doing his best in all contests. He wants to calculate the maximum extra point.

## Input

The input is started by an integer $0<t \leq 10$ followed by $t$ test cases. Each test case is represented by two lines. In the former, a positive integer N is given as the number of contests (i.e. $1<\mathrm{N} \leq 8,000$ ). In the latter, N space-separated integers have given such that the $\mathrm{i}^{\text {th }}$ number indicates the number of problems the student can solve in the $\mathrm{i}^{\text {th }}$ contest. Each contest has exactly 100,000 problems and there is enough time to solve all of them for an intelligent student.

## Output

For each test case, output in a separate line, the maximum extra point.


| Sample Input | Sample Output |
| :---: | :---: |
| 4 | 8 |
| 2 | 6 |
| 35 | 37 |
| 2 | 30 |
| 53 |  |
| 4 |  |
| 510620 |  |
| 10 |  |
| 10987654321 |  |



## F- Phone System

Time limit: 3 Secs
Reading a large number on the phone for others, is one of challenging (!) tasks. We have an interactive voice response (IVR) phone service. To have a better recognition of your number, you are recommended to do the following instructions in order:

- Read only single or two digit numbers
- You cannot read OX as a two digit number
- Read minimum number of zeros (Sefr in Persian)
- And finally read minimum number of single-digit numbers

We want to know the number of possible ways that you can read the given number for our system following our recommendations?

## Input

The first line of the input contains an integer $T \leq 100$ denoting the number of test-cases. Each test-case contains a number string consists of at most 200 digits.

## Output

For each test-case, output on a single line the number of possible ways that you can read the numbers. Since the result may become very large, output the result modulo 1,000,000,007.

| Sample Input | Sample Output |
| :--- | :--- |
| 3 | 1 |
| 000 | 1 |
| 01020320 | 3 |



## G- Geometrical Voice

## Time limit: 3 Secs

In Shiraz University, ACM director wants to inform the students about holding the ACM competition. To create a loud voice he uses an amplifier that distributes the sound on a circle with radius R .

In Shiraz University, students are strange. Whenever they hear some news about anything, they yell it instantly and the voice is distributed on a square with the center of their body. Before shouting, a special sense says them how much loud they should shout. The louder shouting, the larger size of the shouting-square will be achieved. The Length of the edges of the squares is always an integer number and the same for all the students. Each student is fixed in her place and the location of the amplifier is also given and fixed.

By having $R$, you should make a special sense for the students about the minimum size of their shouting-square such that at least $k$ students can hear the news.

## Input

There are multiple test cases. For each one, the first line contains three positive integers N (not greater than 10,000 ), R and $\mathrm{k} \leq \mathrm{N}$, which indicate the number of students, the radius of the voice-circle of the amplifier and the minimum number of students, which should hear the news, respectively. Then in $N$ following lines, each line contains two integers $X_{i}$ and $Y_{i}$ to represent the position of a student. The last line contains two integers $X_{A}$ and $Y_{A}$ representing the position of the amplifier. The absolute value of any integer in the input file will not exceed $1,000,000,000$. The voice of the amplifier reaches to at least one student.

## Output

For each test case, output the length of the edge of the sensed square in a separate line.


|  | Sample Input | Sample Output |
| :--- | :--- | :--- |
| 3 | 3 | 3 |
| -1 | 1 | 6 |
| 1 | -1 | 4 |
| 2 | 2 |  |
| 4 | -1 |  |
| 6 | 2 | 6 |
| 0 | 2 |  |
| -2 | 4 |  |
| -4 | 3 |  |
| -4 | 1 |  |
| -4 | -1 |  |
| -2 | -1 |  |
| 0 | 0 |  |



## H- Hitler's Board

## Time limit: 7 Secs

Adolf Hitler uses a large board-map in the center of "control and command" in his house for managing the resources in the wars. He has encountered a problem now. Let's listen to him:

Hitler: "I have a big board with some nails on it. Consider all distinct pairs of the nails. Each pair can be connected by a string directly. I desired to connect some pairs with the minimum usage of string such that we have a direct or indirect connection between each pair of the nails. Anyway, I did not it yet.

If two nails are connected by a string, it is hard to open it. My wife called me (by phone) and told me some bad news. My son (in my absence) has connected two nails to each other and it means that the minimum usage of string may change. I do not know which pairs are connected now. Corresponding to each pair, we have an extra usage of string. How many of the pairs have the minimum usage of string and what is the expected (Average) extra usage of string."

## Input

The first line of the input contains an integer $\mathrm{T} \leq 100$ denoting the number of test cases. Each test case starts with an integer $2 \leq N \leq 5,000$ denoting the number of nails followed by N lines each containing a pair of integers $x$ and $y$ denoting the position of a nail on the board $0 \leq x, y<5,000$.

## Output

For each test-case, output in a single line the number of pairs with the minimum usage of string followed by a space and then the expected (average) extra usage of string with exactly four digits after floating point.


|  | Sample Input |  |
| :--- | :--- | :--- |
| 2 |  | 40.1381 |
| 4 |  | 82.5171 |
| 0 | 0 |  |
| 1 | 0 |  |
| 0 | 1 |  |
| 1 | 1 |  |
| 8 |  |  |
| 0 | 0 |  |
| 0 | 8 |  |
| 8 | 0 |  |
| 8 | 8 |  |
| 2 | 2 |  |
| 6 | 2 |  |
| 6 | 6 |  |
| 2 | 6 |  |



## I- Intervals

## Time limit: 2 Secs

There are a lot of lamps in Mollasadra street. Each lamp has a height $h$ and a coordinate $x$. Babak wants to find the interval (x1, x2) with the maximum lightness (excluding x1 and x2). An interval has a more lightness if there are more lamps to lighten it. Each lamp lightens a symmetric range around x as long as $\frac{h}{2}$ from each side. You need to find the number of the intervals with the maximum lightness.

## Input

There are multiple test cases. Each one begins with an integer $n \leq 1000$ as the number of lamps followed by $n$ lines. The $i^{\text {th }}$ line contains two integers $x_{i}$ and $h_{i}$ denotes coordinate and height of $\mathrm{i}^{\text {th }}$ lamp (absolute value of each integer is less than $10^{9}$ and $\mathrm{h}_{\mathrm{i}}>0$ is even for each i ).

## Output

For each test case, print two integers. The number of intervals with the maximum lightness and the number of lamps lighten each one.

|  | Sample Input |  | Sample Output |
| :--- | :--- | :--- | :--- |
| 2 | 1 | 2 |  |
| 2 | 4 | 2 | 2 |
| 3 | 6 |  |  |
| 3 |  |  |  |
| 1 | 2 |  |  |
| 2 | 4 |  |  |
| 3 | 2 |  |  |



## J- Julia Composite Subset

## Time limit: 10 Secs

Julia is a teacher and wants to teach her students how to compute greatest-common-divisor (gcd) of two integers. Hence, she desires to have a good subset as a set of integers, none of which are relatively prime (e.g. $\{12,15,18,20,30\}$ ). She starts from $S$ as a candidate set of positive integers. Then for each sequence $\left\langle x=y_{1}, y_{2}, \ldots, y_{2 k+1}\right\rangle(k>0)$, the number $x_{1}$ is thrown out if following conditions are held:

- $y_{i}$ and $y_{i+1}$ are relatively prime
- $x$ and $y_{2 k+1}$ are also relatively prime
- x is smaller than all $\mathrm{y}_{\mathrm{i}}>1$

In other words, from each odd size cycle of integers, in which the adjacent ones are relatively prime, the smallest one is thrown out. This process is continued until there is no such sequence. This filtering is done from the largest $\mathbf{x}$ to the smallest one. For $S=\{1,2,3,4,5,12,15,18,20$, $30\}$, considering the sequences $\langle\underline{3}, 4,5\rangle$ and $\langle\underline{1}, 2,5\rangle$, in order, leads to removing 3 and 1 , respectively.

The good subset is then chosen from the remained numbers. Given $S$, find the maximum possible good subset after the filtering. For the above example, from the set of remained numbers, $\{2,4,12,18,20,30\}$ can be extracted as the largest good subset.

## Input

In the first line, an integer $\mathrm{T}<20$ is given as the number of test cases. Each test case is represented by a line containing a positive integer $n \leq 1000$ as the size of $S$ followed by a line containing n integers as the members of S .

## Output

For each test case, output two separated integers A and B in a line, whereas A is the size of the set after initial filtering and $B$ is the maximum size of a good subset can be finally extracted.


SR

| Sample Input | Sample Output |
| :---: | :---: |
| $\begin{array}{\|llllllllll\|} \hline 3 & & & & & & & \\ 10 & & & & & & & \\ 1 & 2 & 3 & 4 & 5 & 12 & 15 & 18 & 20 & 30 \\ 4 & & & & & & & \\ 12 & 14 & 15 & 20 & & & & \\ 6 & & & & & & & \\ 12 & 14 & 15 & 20 & 23 & 29 & & \\ \hline \end{array}$ | $\begin{array}{ll} \hline & 8 \\ 4 & 3 \\ 2 & 1 \end{array}$ |



## K- Kings-Game

## Time limit: 3 Secs

Design of a game for playing two kings with each other is not easy. The game should be related to their belongings :). A two-king-player game has been designed on a one dimensional board the same as an array of integers. Regardless of how the game is played, each king has finally a pin on a cell. The first king can use all the numbers from the start of the array until his pin and the second king can use the numbers from his pin until the end of the array (one cell may be used by both). The winner is the one who achieve more distinct numbers from his cells. In order to prevent any war, the game should be ended in draw. Can you compute the number of draw states? An state is presented by $(P, Q)$ where $P$ is the index of the first king's pin and $Q$ denotes the index of the other one. For example, consider zero-indexed array $\mathrm{A}[0 . .5]=3,5,7,3,3,5$.

There are exactly fourteen draw states:

$$
(1,4),(1,3),(2,2),(2,1),(2,0),(3,2),(3,1),(3,0),(4,2),(4,1),(4,0),(5,2),(5,1),(5,0)
$$

## Input

Each test case starts with $N \leq 100,000$ as the size of the board. On the next line $N$ integers are given as the members. The numbers are guaranteed to fit in a 32 bit signed integer. End of the input is indicated by $\mathrm{N}=0$.

## Output

For each test case, output the number of draw states on a single line. Since the result may be very large, print it modulo 1,000,000,007 (i.e. $10^{9}+7$ ).

| Sample Input | Sample Output |  |
| :--- | :--- | :--- |
| 6 | 14 |  |
| 357335 |  | 5 |
| 12 | 1 |  |
| 266134456441 |  |  |
| 1 |  |  |
| 1 |  |  |

